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Dividing and Composing the Squares

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ABSTRACT

Geometrical/ artful patterns are valuable in visualization and understanding geometrical/ mathematicalconcepts. In this article, I explain some patterns and methods of dividing and composing squares, based on Abu'l- Wafa Buzdjani's book (4th Hejira/ AD 10th century). In addition, I applied activities as a workshop format in high school geometry classroom. Participants were 14 students grade 11 at a High school in Isfahan. They cut and assembled squares by scissor and painted them. Then they found some applied patterns in geometrical based tiling in Islamic constructions in Isfahan. Finally, they presented their works in power point format to others. Notation to such patterns and arts helped them to better understanding geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.

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Introduction

The Islamic artisan, faced with a belief system that restricted forms of visual display, demonstrated his celebration of Islam by creating attractive abstract geometrical patterns. These very patterns show the role of geometry in Islamic arts. Numbers also played an important part in association with geometry, giving a personality to a shape; the number four, associated with the square, represented stability.
Abul' Wafa (4th Hejira,10th century AD) describes annotative cut and paste constrictions in his famous book about applied geometry: On those parts of geometry needed by craftsman. The part 9 book he explains methods for dividing and reassembling squares that do not rely on Pythagorean Theorem (also known as the Bride's Chair theorem by Arabia writers). He also worked with artisans and provided solutions to their questions. There is evidence to suggest that this approach to geometry was not restricted to 4th century (10th century AD) Baghdad, but was common throughout the Islamic World. The samples explained by Abul' Wafa, indicate relation between artisans and geometers. The examples explained by Abul' Wafa provide descriptive proofs that make a link between theoretical and applied geometry, particularly useful for artisan. In addition, he seems well informed of links between *number theory* and *geometry*.

Purpose of the Article

The main purpose of my study was to investigate applied methods by Abul' Wafa for dividing and composing squares. In addition, I have attempted to explain such patterns related to squares in attractive tiling and wooden arts in Isfahan as a workshop for my students. Then students could see applied geometry in Islamic arts clearly.
Educationally, since geometry is so appealing, we can use the subject to motivate students to take more interest in mathematics and particularly geometry learning. This is very important for increasing and fostering the creative power of students. In addition, searching historical fundamentals through manuscripts and Islamic constructions increased their self-dependence.

Methods for Dividing and Composing Squares

Abul' Wafa classified numbers in two class, *squares* and *non- squares*. In addition, square numbers classified in two classes: **First**, those that are equal to the sum of two other square numbers, such as, $13 = 9 + 4$ or, $41 = 16 + 25$; **Second**, those that are not equal to the sum of two squares such as seven. Where numbers are squares or equal to two squares, changing it to smaller squares is very simple.
Here, I explain Abul' Wafa methods through activities suitable for the classroom:

Activity 1) Divide squares with two and three lengths side to some congruent squares. (Figure1)

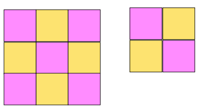


Figure 1: *dividing squares.*

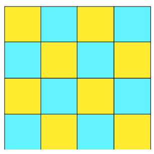


Figure 2: *composing 16* congruent *squares.*

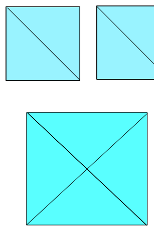
Activity 2) Compose 16 congruent squares to a square. (Figure 2)

Activity 3) For squares that are equal to the sum two squares, make a square.

a) If two squares are equal:

Activity 3-1) we want make a square from two 1×1squares. (figure 3)

(Since $2=1^2+1^2$ we can divide any 1×1 squares in half along a diagonal; these diagonals become the sides of the new square.)



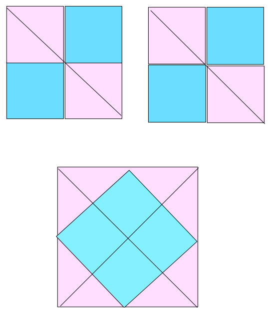


Figure 3: *composing a square from two* 1×1 *square.* **Figure 4**: *composing a square from two* 2×2 *squares.*

Activity 3-2) we want makes a square from two 2×2 squares. (figure 4)

(Since $8=2^2+2^2$ we can divide any of 2×2 squares in half along a diagonal; these diagonals become the sides of the new square.)

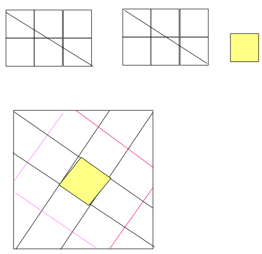
b) If two squares are not equal:

Activity 3-3) $13=4+9=2^2+3^2$

(Since 13=6+6+1 first, divide into two 2×3 rectangles and a unit square, then divide the rectangles along their diagonal, arrange the resulting triangles around the unit square.) (Figure 5)

Activity 3-4) $20=16+4$

(Since 16=8+8+4 first, divide into two 2×4 rectangles and a 2×2 square, then divide the rectangles along their diagonals, then arrange resulting triangles around the square.) (Figure 6)



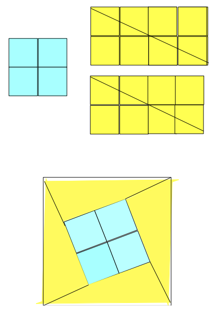


Figure 5: *composing* 2×2 *and* 3×3 *squares.* **Figure 6**: *composing squares.*

The following patterns can result from the above figures:

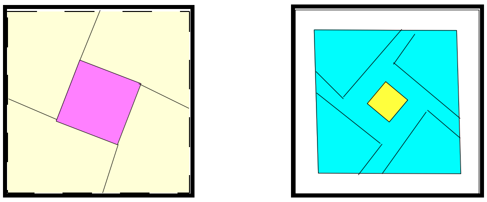


Figure 7: *square patterns.*

Activity 4) Using a non- square number of equal squares to make a square, for example making a square from *three equal* squares. In this section Abul' Wafa explained that artisans made errors when applying this problem. He investigated their methods then rejects some solutions, and provided better solutions for this problem.
First, we cut two squares in half along their diagonals. Then we will lay any part on the sides of the third square sides as figure 8:

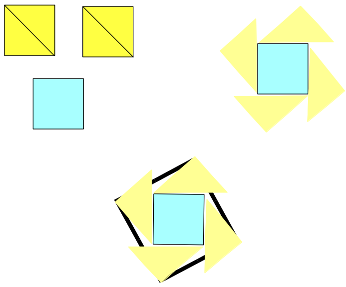


Figure 8: *composing three square to make a 'square'.*

Activity 5) Using some unequal squares to make a square. This method applies generally then when we use two squares and to generate others. First, we will lay the smaller square on the other square. Then cut increased parts from larger square. We will cut a new small square and two rectangles and divide along their diagonal, and then cut those and arrange the resulted part as figure 9:

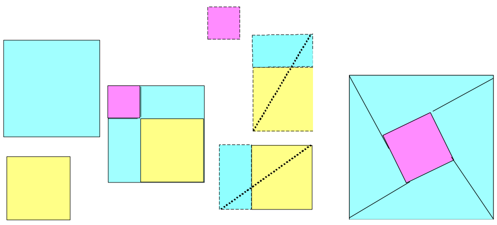


Figure 9: *composing two non- congruent square to make a square.*

Activity 6) we want cut a square with certain side length from another square. First, draw semi-circles with square side length and then by drawing arcs with radius equal to the desired square side mark points on the arcs. Then we will join these points. The resulting square is smaller than desired square. Now, we will cut these parts and make a rectangle from any two triangles and then cut small square from one rectangle and arrange those as figure 10:

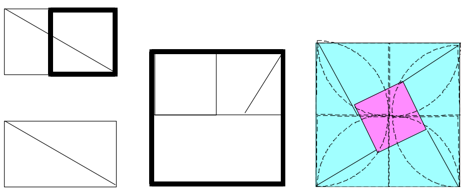


Figure 10*: cutting a square from a one other square.*

The following pattern is from an *applied geometry* book. It is from an appendix written several centuries after Buzjani (Jazbi believes the writer is Koobnaani, but Alpay believes the author is unknown). Consider squares and rectangles.

Activity 7) In this section, students have provided some artful samples by following pattern. By survey tiling, wooden arts and other Islamic arts in Isfahan we found very attractive samples.

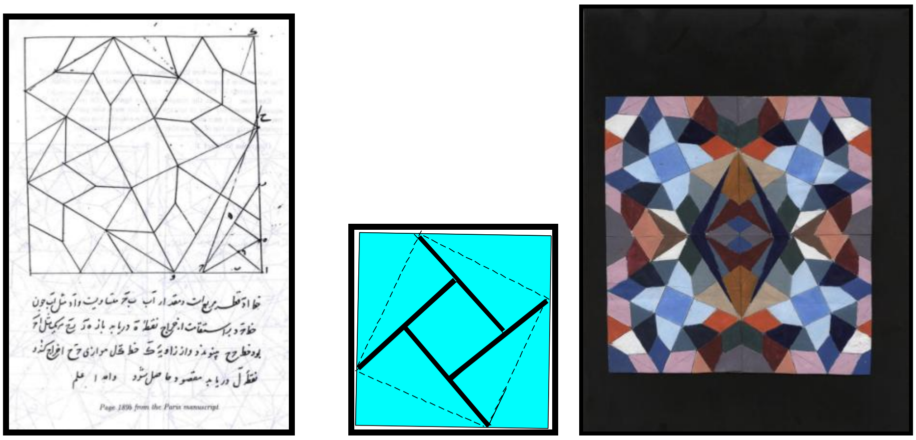


Figure 11: *a problem from unknown author and samples of student works.*

Samples of geometrical patterns in Isfahan constructions

Finally, here are some illustrations of applied square patterns that the students are able to see around them in Isfahan.

Isfahan Friday mosque: The date of building construction referred to 4^{th} to 11^{th} hejira (AD10th - 17th) century.

Figure 12: *North Iwan, four symmetric divided square.*

These tiling have applied pattern figure 8.

Figure 13: *An attractive square tiling, south Iwan.*

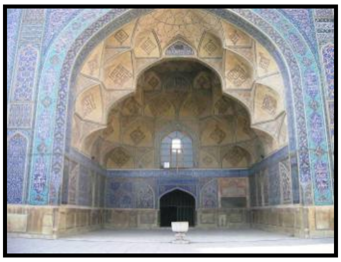


Figure 14: *south Iwan perspective.*

Here are some more four squares tilings.

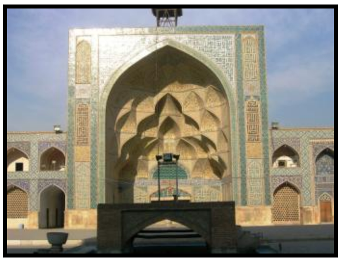


Figure 15: *west Iwan perspective.*

Figure 16*: west Iwan, and square tiling.*

Figure 17*: west Iwan, and square tiling.*

Figure 18*: west Iwan, and square tiling that covered by Ali name.*

In the next picture, you can see a geometrical shapes that covered by poem that known as *katibeh khate bannaee* and consist of a poem about Imam Ali:

Figure 19: *katibeh in west Iwan*.

Shikh lotf ol lah mosque: this construction built in 11th hejira (AD 17th century) by order of *Shah Abbas Safavi*. One of archaeologists said: ' *hardly, I can say that this building has been constructed with human hands'.*

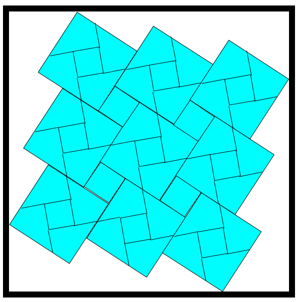


Figure 20: *repeated pattern.*

Figure 21: *wooden door covered by repeated pattern.*

Khan mosque: the date of building khan mosque referred to 1090 hejira (AD 17th century) by order of *Shah Soleiman Safavi.* The brick tiling constructions in khan mosque is very attractive and unique.

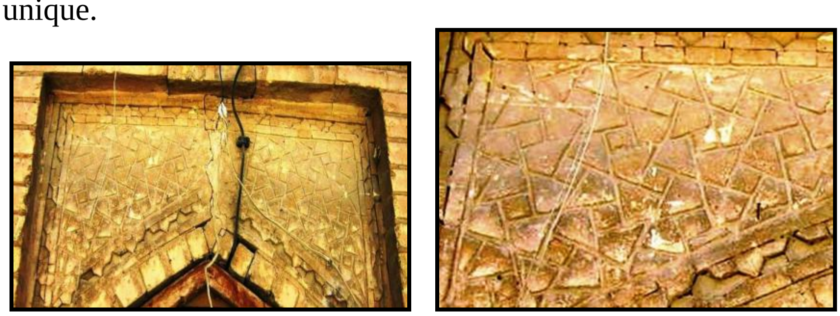


Figure 22: *brick art repeated pattern.*

Jme'e Abbasi mosque: building mosque started in *Safavi age* (1020 hejira/ 17 AD century).

Figure 23: *square tiling in jame'e Abbasi mosque.*

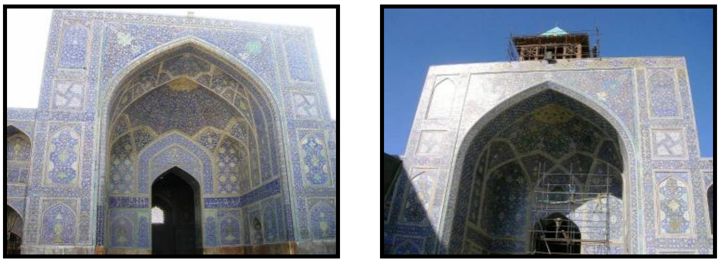


Figure 24: *symmetric square tiling in west and east side of jame'e Abbasi mosque.*

Chahar bagh madrasa: the beautiful building in Shah *Soltan Hosein Safavi* (1116-1126 hejira/AD 17th century).


Figure 25: *symmetric square tiling in chahar bagh madrasa.*

Concluding Remarks

In conclusion, the results of this research indicate, by implication, that with a historical background in mathematics education, particularly geometry, the learner can make links between the arts and mathematics. Our students generally ask where is mathematics used, then we can state and analyze such valuable samples to improve their mathematical thinking, and so foster their creativity power. By using these patterns, we will teach abstract subjects as applied subjects. In addition, by using the geometrical patterns, students could have a better understanding about geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.

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