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Analysis of the sundials on the "Tower of the Winds" in Athens
(First results based on new data attained with a high precision geodetic data acquisition system)

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The "Tower of the Winds" in Athens, built by Andronikos Kyrrhestes, is one of the best preserved monuments from ancient times. In the past the sundials on the Tower of the Winds have been analyzed by Delambre (1817), Drecker (1925), Antonacopoulos/Fragakis (1969) and Bromley/Wright (1989) based on the measurements done in 1762 by J.Stuart and N.Revett. Only Palaskas (1846) performed own measurements. The measurements in all these cases as is apparent now from the new measurements are not that accurate as they seem to be. Despite that all authors draw conclusions about the accuracy of the construction in a favourable way. Gibbs (1976) seems to be the first to having recognized the few remains on the cylindric annex as part of another sundial

It may be noticed, that Vitruvius mentions the "Tower of the Winds" and its designer (book I,25) only in the context of finding the cardinal directions for town planning but not the Horologion. Again later (book IX) when he deals with sundials in particular no mention is done about this outstanding time standard. This aroused a discussion (Delambre) about wether the sundials were added later. Palaskas objected to this already, and if it is not already clear from the overall architectural concept, the design of the cylindrical sundial would be proof enough.

The current analysis of the sundials is part of a Greek-German-Project (Kienast) to thoroughly investigate the whole building. The data acquisition was performed with the Leica System TC 1610 resulting in absolute coordinates with an uncertainty of less than 3 mm. The position and orientation of the coordinate system is also known to high precision.

Taking into account the achievements in mathematics, astronomy and geography available at that time the analysis shall reveal the cardinal design parameters of the sundials as they are

- geographic latitude and orientation of the wall
- angle of ecliptic,
- length of the gnomon in ancient units.

It shall be pointed out here, that the "engineers" of that time faced a similar problem of metrology as must still be solved today in the national laboratories: When setting up a primary standard - the atomic clock nowadays - there is no simple standard available to which to compare. The duration of one day, when the shadow of the sundial arrived at the very same position (which is observable with an uncertainty of only a few seconds), could have been used for calibrating the water-clock.

Since the objective of the analysis is to find parameters used for the construction of given sundials, not to construct a sundial according to given parameters, one is faced with an "inverse problem" with the intrinsic difficulty that it may tend to be "ill conditioned" because of deficiencies in the available data. The "inverse problem" will not be solved in this case with a general approach but rather by knowledge guided trial and error. A set of parameters is selected in order to calculate a model which will be compared with the reality presented. The consequence of the deficiencies will be that two or more models are equivalent according to certain error criteria. Fortunately the range of parameters is limited by the nature of the problem and by practical considerations of the design. A final selection will take advantage of special points on the day line at equinox and the hour line at noon together with the apriori knowlegde about the parameters.

There are mainly four reasons for the deterioration of the data:

- deficiency of the basic concept,
- deficiencies during the construction of the sundials,
- changes in time by external impact (earthquakes),
- uncertaincies of the data acquisition.

The high precision measurements showed that the walls to the South, South-East and East are still very well in place after the long period and add no problems to the analysis. Doubts about the basic concept were listed merely for completeness, thus the analysis for the three sundials mentioned above will have to deal only with the imperfections of the realization.

The first cardinal elements - position of the wall and its orientation with respect to the cardinal points. - have been verified. The position of the tower was now found with respect to two trigonometric points as:

$$37^\circ 58' 21''.6 \ \mathrm{N} \quad ; \quad 23^\circ 43' 28''.8 \ \mathrm{E}$$

The orientation of the tower has been determined by observation of the sun and by utilizing information from the global positioning system (GPS). A coordinate system was defined for the tower based on the best fit of a regular octogon to the inner of the tower at floor level. The cardinal axis of this octogon nearest to the Nord-South-direction has an Azimut of $0^\circ 05' 30''$.

Palaskas, who determined the orientation of several faces outside obtained a mean value of 17' which shows the imperfection of his measurements not, as he claims, Andronikos', who mostlikely used the method now known as "Indian circles" as described by Vitruvius (book I,6) obviously to great perfection.

The angle of the ecliptic was known in ancient time with limited accuracy: $\epsilon = 23^\circ 51'$ (Eratosthenes) . For practical reasons of geometric constructions the value $\epsilon = 24^\circ$ (Vitruvius) might have been used. The value actually changes slowly with time so as to be $\epsilon = 23^\circ 42' 20''$ in 100 B.C. ($23^\circ 26' 21''$ in 2000 A.C.).

Summary of cardinal elements of the sundials

	gF unciae	gF mm	tan φ	φ ° ' "	ϵ ° ' "
S	11	271.5		37 58 15	23 42 20
SO	30 *	740.4		37 58 15	23 42 20
O	20	493.6	4/5	38 39 35	24 00 00
NO	9 *	222.1		37 58 15	23 42 20
N	11	271.5		37 58 15	23 42 20
NW	9 *	222.1		37 58 15	23 42 20
W	20	493.6	4/5	38 39 35	24 00 00
SW	30 *	740.4		37 58 15	23 42 20

* angle against normal of the wall : 30°

Finally regarding the cylinder dial Bromley and Wright already pointed out that the remaining lines on the cylinder cannot be explained with gnomons and call for another solution. The following proposal has convincing aspects. Fig. 2a shows how the combination of the shadow cast by the cornice like a "brim" of a "hat" together with the "eigen-shadow" of the cylinder body forms a corner point which is unique during the day and the seasons. Traces on the South-face (Fig. 2c) lead to the reconstruction of a cornice of 8 unciae depth. With the values (φ = arctan(4/5) , ϵ = 24°) this goes well together with the position of the lower end of the noon line on the cylinder at summer solstice. Unfortunately the theory demands that the day line crosses at a right angle with the hour line at noon. The "day line" might also be interpreted as the shape of the shadow at noon, reducing the sundial to a noon indicator. In this case the theory would call for a much smaller angle towards the vertical. Since there is no other functional requirement for the corcice regarding the architecture of the annex the basic idea - the corner point of the two shadows - should be correct.

These differences have only small effects on the shape of the sundials, and one can certainly not hope - as Palaskas suggested - to date the building this way.

The geographic latitude had been determined as the ratio of the shadow length (equinox noon) to gnomon height, expressed with small integer numbers. The following values were reported in case of Athens (with modern writing) :

φ = arctan(3/4) = 36° 52′12" Vitruvius (book IX,7)
φ = arctan(16/21) = 37°18′14" Plinius (book VI,211)

With the East- and West-sundials the angle of the day line at equinox against the vertical directly represents the latitude. It is found here close to

φ = arctan (16/20) = 38° 39′35"

Finally the length of the gnomon is the free parameter to be chosen by the designer for essentially determining the overall size of the sundial. One should reasonably expect "nice" values in terms of the ancient unit of length. As a hypothesis supported by several dimensions of the tower itself it may be assumed that the construction is based on the roman foot:

1 pes monetalis = 16 digiti = 12 unciae = 296.17 mm

This implies that the tower was built after this length standard had been established in Athens by the Romans. If this result can be confirmed this investigation would also help to answer the question (Freeden) about the time - late 2nd century B.C. or early 1st century B.C. - when the tower was built.

In contrast to modern sundials having a stylus mounted parallel to the axis of the globe [OQ] (Fig.1), only the shadow of the gnomon tip is essential for ancient sundials thus giving freedom for how to mount the gnomon onto the wall. In fact the gnomons were mounted here above the horizon and the length is to be measured normal to the wall [OF] in case of S-, N-, E- and W-dials whereas for the others there is an angle of 30° [OF_2] to the normal (Palaskas is wrong with the NE- / NW-dial giving 45°).

Possibly it can be decided about the question wether the sundials have been constructed according to the theory with certain parameters - as most authors suppose - or just by marking the shadow using another "horologion", maybe the water-clock inside, as a standard (Palaskas). It is remarkable in particular that different values of latitude seem to be implemented. From this it appears plausible that the combination (φ = arctan(4/5), ϵ = 24°) is to be interpreted as geometrical construction, whereas the actual values (37°58'22" / 23°42'20") give a hint to the empirical method. The summery of parameters lists the values of the current working hypothesis.

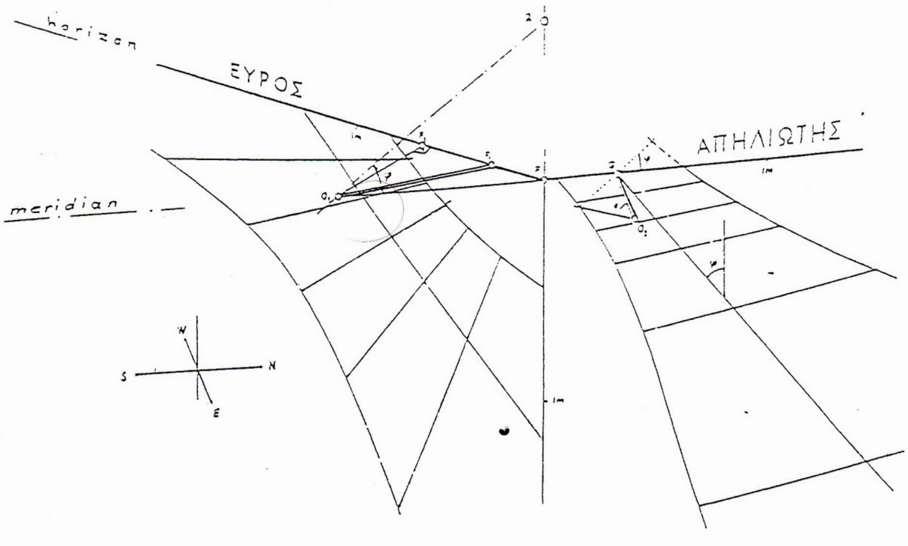

Fig. 1 Partial view of the sundials at the South-East and East-face showing different cardinal parameters

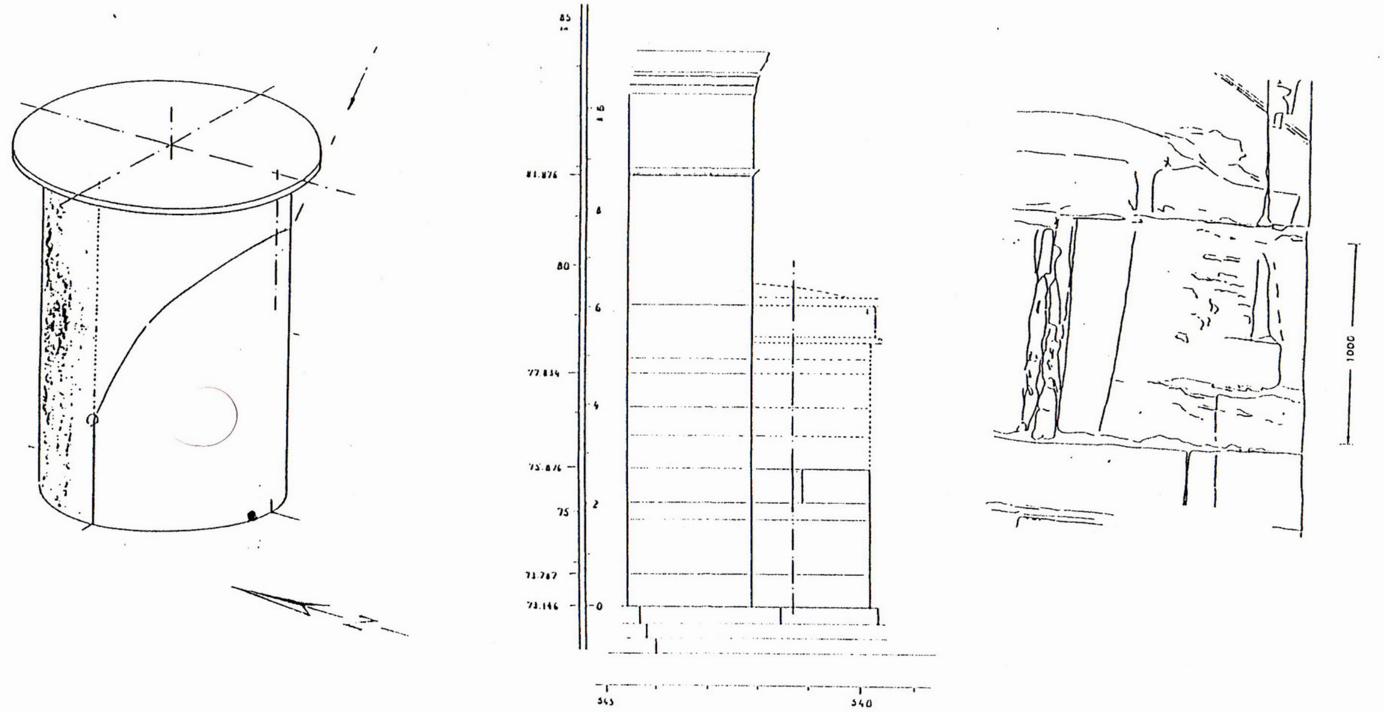

Fig. 2 Sundial on the cylinder of the annex

a) functional principle

b) general view from the West

c) traces of the annex on the South-face of the taower

Bibliography

[1] C.Fensterbusch - Vitruv , Zehn Bücher über Architektur,
übersetzt und mit Anmerkungen versehen
Wissenschaftl. Buchges., Darmstadt (1964), (1991) 5.th ed.

[2] J.Stuart, N.Revett - The antiquities of Athens
London (1762) , 2nd. ed. (1825)

[3] M.Delambre - L'histoire de l'astronomie ancienne
vol.II, chap.16 De l'Analemme
Paris (1817)

[4] L.Palaskas - Mémoire presentée à la Société Archéologique d'Athènes, Le 15 Novembre 1845
Συνοψης των πρακτικων της αρχαιολογικης εταιριας των Αθηνων (Εκδοσις δευτερα)
Resumé des actes de la sociéte archéologique d'Athènes
(1846) p.247-286

[5] J.Drecker - Die Theorie der Sonnenuhren
in: E.v.Bassermann-Jordan (ed.) - Geschichte der Zeitmessung und der Uhren
W.de Gruyter Verlag, Berlin, Leipzig (1925) Bd.1, Lfg.E

[6] G.Antonacopoulos, C.Fragakis - The Andronikos Horologium in Athens (Tower of the Winds)
Γρ. Αντωνακοπουλος, Χ. Φραγκακης - Το εν Αθηναιας ηλιακον ωρολογιον του Ανδρονικου
AAA, vol.2 (1969) p.416-422
(AAA · Αρχαιολογικα Αναλεκτα εξ Αθηνων)

[7] Sh.Gibbs - Greek and Roman Sundials
Yale University Press, New Haven, London (1976)

[8] J.v.Freeden - ΟΙΚΙΑ ΚΥΡΡΗΣΤΟΥ ,
Studien zum sogenannten Turm der Winde in Athen
(1983)

[9] A.G.Bromley, M.T.Wright - The Temporal Hours of the Tower of the Winds at Athens
Basser Dept. of Computer Science, University of Sydney
Technical Report No.365, (1989)

[10] H.Kienast - Untersuchungen am Turm der Winde
Archäologischer Anzeiger (1993) p.271-275

[11] H.Kienast - Antike Zeitmessung auf der Agora
(Neue Forschungen am Turm der Winde in Athen)
Antike Welt vol.28 (1997) no.2, p.113-115